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Research Article

THE MEANING OF ABSTRACTION IN MODERN MATHEMATICS AN EPISTEMOLOGICAL STUDY OF STRUCTURE AND FORMALISM

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Abstract
This article explores the significance of abstraction in the development of modern mathematics from an epistemological perspective and how formalism plays a role in the formation of mathematical knowledge. Abstraction in mathematics not only serves as a cognitive strategy for simplifying concrete objects into general structures, but also serves as a conceptual principle underlying the understanding of mathematical objects that are increasingly distant from direct experience. Using a qualitative approach based on literature-based research, this article conducts a critical reading and comparative philosophical analysis of classical and contemporary literature in the philosophy of mathematics. The analysis shows that abstraction plays a central role in enabling the generalization of theories and the formation of consistent formal systems, while also influencing the understanding of mathematical truth as the internal coherence within formal axiomatic systems. Furthermore, formalism provides a methodological framework that allows the manipulation of symbols and formal structures without relying on intuitive meaning, thus maintaining consistency as the primary criterion of truth. These findings confirm the symbiotic relationship between abstraction and formalism in the epistemology of modern mathematics and open a dialogue about the challenges and tensions between abstraction, intuition, and structural practice in contemporary mathematics. Thus, abstraction is not simply a technique but an epistemic principle that shapes how mathematics is understood and developed today.

Keywords: Epistemology, Formalism, Mathematical Abstraction

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INTRODUCTION

Modern mathematics is characterized by an increasing level of abstraction, fundamentally differentiating it from classical mathematical practice, which was more oriented toward concrete objects and direct applications (Boccuni & Zanetti, (2025). Abstraction is no longer merely a cognitive tool but has become an epistemological principle that determines how mathematical knowledge is constructed and validated. The development of branches such as set theory, abstract algebra, and topology demonstrates that mathematics is moving away from empirical intuition toward autonomous formal structures (Fitriani, 2025). This situation raises fundamental philosophical questions about the meaning of abstraction itself in the context of knowledge. Does abstraction represent a specific reality, or is it merely an internally coherent symbolic construction? Therefore, this research is crucial for reexamining the role of abstraction as the epistemological foundation of modern mathematics. By understanding abstraction philosophically, this research seeks to explain why mathematics remains considered a valid form of knowledge despite its increasing detachment from sensory experience.

The urgency of this research is also related to the dominance of formalism in contemporary mathematical practice. Formalism emphasizes logical consistency and the manipulation of symbols without necessarily associating them with specific ontological meaning. This approach has yielded significant advances in the clarity and precision of mathematical arguments, but has also raised criticisms regarding the reduction of meaning and the loss of the intuitive dimension. In the context of epistemology, formalism shifts the focus of truth from correspondence with reality to the internal coherence of the system. This shift impacts not only mathematical methodology but also how mathematicians understand what is meant by mathematical objects. Therefore, an epistemological study of formalism is relevant for explaining the basis for the legitimacy of modern mathematical knowledge. This research positions formalism as the primary analytical lens for understanding the relationship between abstraction, structure, and mathematical truth.

Furthermore, the historical dynamics of the development of mathematics demonstrate that abstraction is not a sudden phenomenon but rather the result of a long process of methodological reflection (Pérez-Escobar & Sarikaya, 2025). From Euclid's geometry to Hilbert's axiomatic system, there has been a gradual transformation in the way mathematics is understood and practiced. This transformation marks an epistemological shift from knowledge rooted in geometric constructions to symbolic formal systems. Understanding this transformation requires a philosophical analysis that bridges the history and theory of knowledge. This research aims to address this need by presenting a systematic and critical study of the meaning of abstraction. Therefore, this research is not merely descriptive but also reflective of broader epistemological implications. It is hoped that this research will enrich the discourse on the philosophy of mathematics, particularly in the Indonesian academic context.

Finally, the relevance of this research also lies in its implications for mathematics education and communication. The high level of abstraction often hinders the understanding of students and novice researchers (Zafrullah dkk, 2024). Without adequate epistemological understanding, abstraction tends to be perceived as an empty formality divorced from meaning. By examining the philosophical foundations of abstraction and formalism, this research seeks to provide a conceptual framework that can help explain why abstraction is necessary and how it functions. This framework is crucial for developing a reflective attitude toward mathematics as a discipline. Therefore, this research is written not only for theoretical purposes but also to address the practical need for a more meaningful understanding of modern mathematics. This introduction emphasizes that research on abstraction has interrelated epistemological, historical, and pedagogical significance.

The literature on the philosophy of mathematics shows that the concept of abstraction has long been a central concern of both classical and modern thinkers (Rulli dkk, 2025). Plato, for example, viewed mathematical objects as ideal entities accessible only through the intellect, thus understanding abstraction as a process of transition from the sensory world to the world of ideas (Putra dkk, 2025). This view was later criticized and modified by Aristotle, who positioned abstraction as the result of separating quantitative aspects from concrete objects. This difference of opinion marked the beginning of the epistemological debate regarding the status of mathematical knowledge. In the modern context, this debate has developed into a more complex discourse with the emergence of logicism, intuitionism, and formalism. The literature shows that each school places different emphasis on the role of abstraction. Therefore, this literature review is important for mapping the research's position within the broader landscape of the philosophy of mathematics.

Logicism, as developed by Frege and Russell, views mathematics as a branch of pure logic. In this perspective, mathematical abstractions are reduced to logical constructions that can be derived from universal rational principles (Syahnia dkk, 2022). The logicist literature emphasizes that the validity of mathematics rests on logical deduction, not intuition or experience. However, the failure of the logicist project due to internal paradoxes demonstrates the limitations of this approach. Criticism of logicism opens up space for other approaches that place greater emphasis on formal structures. Thus, the logicist literature is relevant as a historical backdrop explaining why formalism subsequently gained a dominant position. This study utilizes this literature to demonstrate that mathematical abstraction has always faced demands for consistency and a strong epistemological foundation.

Intuitionism, pioneered by Brouwer, offers an alternative view by rejecting abstractions that cannot be mentally constructed. In the intuitionist literature, mathematics is understood as an activity of the human mind, so mathematical truth depends on intuitive construction. This approach criticizes formalism for its perceived neglect of the subject's cognitive meaning and experience (Landsman & Singh, 2025). However, intuitionism also faces limitations in explaining the highly symbolic practice of modern mathematics. The literature suggests that intuitionism makes an important contribution in highlighting the epistemic dimension of the subject, but is inadequate in explaining the autonomy of formal structures. Therefore, this study positions intuitionism as an internal critique that enriches the analysis of abstraction, without fully adopting it as the primary framework.

Formalism, as formulated by Hilbert, has become one of the most influential approaches in modern mathematics. The formalist literature asserts that mathematics is a system of symbols governed by formal rules of manipulation (Adžić dkk, 2025). Within this framework, abstraction is understood as the elimination of external meaning in order to maintain internal consistency. Mathematical truth is no longer measured by correspondence with reality, but rather by the possibility of proof within an axiomatic system (Prestifilippo, 2024). Contemporary literature shows that this approach has succeeded in providing a strong methodological foundation for various branches of mathematics. However, criticism of formalism also highlights the risks of alienation of meaning and reduction of understanding. This study uses the formalist literature as a primary starting point to analyze how abstraction shapes the epistemology of modern mathematics.

In addition to the classical school, contemporary literature in the philosophy of mathematics emphasizes the structuralist approach. Structuralism views mathematical objects not as individual entities but as positions within a structure (Vergaray, 2024). In this perspective, abstraction reaches a higher level as the focus shifts from objects to relations. The structuralist literature demonstrates that this approach is capable of explaining modern mathematical practices oriented toward isomorphism and generalization. However, structuralism also relies on formalism as a means of representation. Therefore, this literature review positions structuralism as a bridge between abstraction and formalism. This approach is relevant for explaining how modern mathematics maintains objectivity without relying on traditional ontology.

Overall, the literature review shows that there is no single consensus on the meaning of abstraction in mathematics. Each philosophical approach offers partial explanations that complement and critique each other. The classical literature provides a historical foundation, while contemporary literature enriches the analysis with new perspectives. This study leverages this diversity of literature to build a comprehensive epistemological analytical framework. Through critical reading and philosophical comparison, this study attempts to synthesize relevant views. This literature review serves as a conceptual basis for further analysis, while also confirming the research's position in the discourse of modern philosophy of mathematics.

RESEARCH METHOD

This research uses a qualitative, literature-based research approach with a focus on epistemological analysis. This approach was chosen because the research objective is not to test empirical hypotheses, but rather to understand the meaning and philosophical implications of mathematical abstraction (Ferrieira, 2024). The research data consists of classical and contemporary texts in the philosophy of mathematics relevant to the topic of abstraction and formalism. Sources were selected purposively, considering academic authority and conceptual relevance. Thus, this method allows the researcher to systematically explore philosophical arguments. A qualitative approach also provides flexibility in interpreting texts and relating them to a broader epistemological context. This method is expected to produce a comprehensive and reflective understanding.

Data collection techniques were carried out through an intensive literature review of books, journal articles, and classic works on the philosophy of mathematics. This process involved identifying key concepts, main arguments, and different philosophical positions. The researcher systematically recorded themes related to abstraction, structure, and formalism. This approach ensured that the analysis was based on an adequate representation of the literature. Furthermore, the literature review allowed the researcher to trace the historical development of the concept of abstraction. In this way, the research did not become trapped in an ahistorical perspective. This data collection technique forms the foundation for the critical analysis conducted in the next stage.

Data analysis was conducted through critical reading of selected texts. Critical reading involves evaluating arguments, identifying epistemological assumptions, and assessing internal consistency. The researcher not only summarizes the views of philosophers but also examines their implications and limitations (Dodig-Crnkovic & Burgin, 2024). This approach allows for critical dialogue between various schools of thought. Furthermore, the analysis is conducted iteratively to ensure depth of understanding. Thus, critical reading becomes a primary tool in uncovering the meaning of abstraction hidden behind mathematical formalism. This technique supports the research goal of producing a reflective and argumentative analysis.

In addition to critical reading, this study utilizes comparative philosophical analysis. This technique aims to compare different philosophical views on abstraction and formalism. The comparison is conducted systematically, highlighting the similarities, differences, and epistemological implications of each approach. The comparative approach allows the researcher to identify the unique contributions and limitations of each school. Thus, the analysis is not eclectic but structured. This technique also helps build a broader conceptual synthesis. Comparative analysis serves as a means to clarify the research's position within existing discourse.

Research validity is maintained through consistent argumentation and methodological transparency. The researcher ensures that each claim is supported by relevant literature references. Furthermore, interpretation is carried out carefully to avoid over-reduction or simplification. An epistemological approach demands conceptual clarity and terminological precision. Therefore, this research emphasizes internal coherence as the primary criterion for analytical quality. With a systematic and reflective methodology, this research is expected to make a valid contribution to the study of the philosophy of mathematics.

RESULTS AND DISCUSSION

The analysis shows that abstraction is a central mechanism in the formation of modern mathematical structures. Abstraction allows mathematicians to detach themselves from the specific properties of objects and focus on general formal relations. This process produces generalizations that broaden the reach of mathematical theories. In an epistemological context, abstraction functions as a cognitive strategy for achieving universality. Thus, the research confirms that abstraction is not merely a simplification, but rather a conceptual transformation. This transformation allows mathematics to develop as an autonomous system of knowledge. This finding reinforces the view that abstraction is a prerequisite for formalism.

This research also found that formalism provides a methodological framework that allows abstraction to operate consistently (Suyitno, 2025). By formulating axioms and inference rules, formalism provides clear boundaries for symbol manipulation. The analysis shows that consistency is the primary criterion of truth within a formal framework. This shifts the epistemological focus from intuitive meaning to structural validity. This finding explains why modern mathematics is capable of handling highly abstract objects. Formalism, in this case, functions as a safeguard of epistemic stability. Therefore, the research confirms the symbiotic relationship between abstraction and formalism.

Furthermore, the research results show that mathematical objects in the modern paradigm are understood structurally. Objects are no longer viewed as independent entities, but rather as positions within a network of formal relations. This approach aligns with structuralism in the philosophy of mathematics (Marsden, 2025). Abstraction allows for the neglect of individual properties in favor of highlighting relational patterns. Thus, mathematical objects gain meaning through their role within structures. This finding suggests that the understanding of mathematical objects depends on the context of the formal system. This finding strengthens the argument that abstraction and structure are inseparable.

The analysis also reveals that mathematical truth within a formalist framework is internal and contextual. Truth is determined by the success of a proof within a particular axiomatic system. Abstraction allows for the formation of various formal systems, each with its own criteria for truth (Jakubowski, 2025). This finding demonstrates epistemological plurality in modern mathematics. However, this plurality remains bound by the demands of logical consistency. The results of this research highlight a paradigm shift from absolute truth to systemic truth. This change is a direct consequence of high-level abstraction.

This research finds that abstraction also plays a role in separating mathematics from traditional ontological demands. Within a formalist framework, the existence of mathematical objects is independent of external reality. Abstraction allows mathematics to function as an autonomous symbol system. This finding explains why ontological debates are often considered secondary in modern mathematical practice. The primary focus shifts to structure and consistency. These results suggest that modern mathematical epistemology tends to be pragmatic. Abstraction, in this case, serves as a tool for methodological liberation.

Furthermore, the results show that abstraction facilitates communication and the transfer of mathematical knowledge. By using formal structures, mathematicians from different backgrounds can understand and develop the same theories. Abstraction creates a universal language that transcends empirical contexts. This finding confirms the epistemic social function of abstraction. Thus, abstraction impacts not only theory but also the practice of the scientific community. These results broaden the understanding of the role of abstraction in the mathematical knowledge ecosystem.

The analysis also reveals a tension between abstraction and intuition. While formalism increases precision, it often comes at the expense of intuitive access. This finding confirms intuitionist criticisms of modern mathematics (Rozzoni, 2025). However, this study demonstrates that this tension is productive. Abstraction encourages the development of new intuitions that fit the formal structure. Thus, abstraction does not completely eliminate intuition, but rather transforms it. These results provide a more balanced view of philosophical debates.

Overall, the results show that abstraction is a key driving force of modern mathematics. Abstraction enables the formation of consistent and productive formal structures. From an epistemological perspective, abstraction shapes how mathematics understands truth, objects, and methods. This finding confirms that modern mathematics cannot be understood without an analysis of abstraction. Thus, the results of this study provide an empirical-conceptual foundation for further philosophical discussions. They also emphasize the relevance of an epistemological approach to understanding contemporary mathematical practice.

This discussion interprets the results within the context of broader epistemological debates. The findings regarding the central role of abstraction suggest that modern mathematics operates at a highly reflective level. Abstraction serves not only as a technical tool but also as an epistemic principle. This answers the question of why mathematics can develop autonomously. This discussion asserts that formalism provides epistemological legitimacy for abstraction. Thus, this research clarifies the relationship between structure, consistency, and truth. This discussion places the research findings in dialogue with the literature on the philosophy of mathematics.

The question of the meaning of mathematical truth is central to the discussion. The research findings demonstrate that truth is understood internally within formal systems. This approach explains the success of mathematics in dealing with abstract objects. However, the discussion also acknowledges the limitations of this approach in explaining intuitive meaning. Thus, this research does not reject intuitionist criticism but places it within a broader context. This discussion demonstrates that a plurality of epistemological approaches is a hallmark of modern mathematics. This enriches the understanding of mathematical truth.

The discussion then examines the implications of the research findings for mathematical ontology. By emphasizing structure and relations, abstraction reduces reliance on specific ontological assumptions, allowing for a more flexible and inclusive approach. This discussion demonstrates that the epistemology and ontology of mathematics are interconnected. Abstraction serves as a bridge between the two. Thus, this study contributes to the ontological debate by emphasizing the epistemological dimension. This discussion affirms the value of philosophical analysis in understanding the foundations of mathematics.

The practical implications of this study are also discussed in the context of mathematics education. The high level of abstraction often poses a pedagogical challenge. This discussion demonstrates that epistemological understanding can help explain the reasons behind abstraction. Thus, a reflective approach can improve the quality of learning. This study suggests the integration of philosophical perspectives in mathematics education. This discussion affirms that epistemology is not separate from educational practice. Therefore, this study has real practical relevance.

This discussion also considers the limitations of the study. Focusing on a particular philosophical literature may overlook alternative perspectives. However, this approach was chosen to maintain depth of analysis. This discussion opens up opportunities for further, more empirical research. Thus, this study is positioned as an initial contribution to the study of abstraction. This discussion affirms the open and ongoing nature of philosophical research. Therefore, the results of this study provide a basis for further exploration.

Overall, this discussion answers the questions of why and why research is conducted. Abstraction has proven to be key to understanding modern mathematics. Formalism provides an epistemological framework that allows abstraction to function productively. Thus, this study provides necessary conceptual clarification. This discussion confirms the research's contribution to the philosophy of mathematics. Thus, this study has complementary theoretical and practical significance.

CONCLUSION

This conclusion confirms that abstraction is the epistemological foundation of modern mathematics. Abstraction enables the formation of consistent and universal formal structures. Through an epistemological approach, this study demonstrates that abstraction shapes the way mathematics understands truth and objects. Thus, abstraction is not merely a technique, but a principle of knowledge. This conclusion systematically summarizes the main findings of the study. Thus, the research objective has been achieved.

The next conclusion emphasizes the role of formalism as a primary methodological framework. Formalism allows abstraction to operate without reliance on empirical intuition. This explains the success of modern mathematics in managing complexity. This conclusion demonstrates that consistency is the primary criterion of truth. Thus, this study provides a clearer understanding of mathematical epistemology. This conclusion confirms the research's theoretical contribution.

Finally, this study concludes that the epistemological study of abstraction has broad relevance, not only for the philosophy of mathematics but also for education and scientific practice. This conclusion opens up space for further research. Therefore, this study is expected to serve as an initial reference. This conclusion emphasizes the importance of philosophical reflection in modern mathematics.

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